Self-Organization of Sorted Patterned Ground

M. A. Kessler*† and B. T. Werner

Striking circular, labyrinthine, polygonal, and striped patterns of stones and soil self-organize in many polar and high alpine environments. These forms emerge because freeze-thaw cycles drive an interplay between two feedback mechanisms. First, formation of ice lenses in freezing soil sorts stones and soil by displacing soil toward soil-rich domains and stones toward stone-rich domains. Second, stones are transported along the axis of elongate stone domains, which are squeezed and confined as freezing soil domains expand. In a numerical model implementing these feedbacks, circles, labyrinths, and islands form when sorting dominates; polygonal networks form when stone domain squeezing and confinement dominate; and stripes form as hillslope gradient is increased.

Patterns delineated by distinct stone and soil (fine-grained) domains visible at the ground surface are formed by cyclic freezing and thawing of decimeter- to meter-thick soil layers in polar and high alpine environments. The observed range of sorted patterned ground includes sorted circles, labyrinthine stone and soil networks, stone islands, sorted polygons, and sorted stripes on hillslopes (Fig. 1). These quintessential forms constitute one of the most striking suites of geomorphic patterns. The diversity of sorted patterned ground has been attributed to the multiplicity of formation mechanisms (1). The underlying processes include particle sorting (2, 3), freezing and thawing (2, 4, 5), deformation of frozen soil (6), and soil creep (7), but the range of forms has not been captured in a single model (8–11).

Patterns in a broad range of environments have been hypothesized to form by self-organization [e.g., (12–17)], whereby nonlinear, dissipative interactions among the small- and fast-scale constituents of a system give rise to order at larger spatial and longer temporal scales (18). Because transport in the active layer (the soil layer experiencing annual or diurnal freezing and thawing) is highly nonlinear and dissipative, self-organization is a candidate for the general mechanism underlying sorted patterned ground (10–12). In this case, a smooth change in controlling parameters might lead to an abrupt shift in the type of sorted patterned ground without a change in processes causing the pattern. We have developed a numerical model within which sorted patterned ground self-organizes, with transitions between patterns controlled by the relative magnitude of two feedback mechanisms plus hillslope gradient.

The first feedback (Fig. 2), between stone-soil interface morphology and transport of stones and soil by frost heave, acts to laterally sort the active layer by moving stones toward areas of high stone concentration and soil toward areas of high soil concentration. Given a layer of stones overlying fine-grained soil (formed by deposition or vertical sorting), a laterally uniform stone-soil interface is unstable to perturbations because of frost heave near the interface. A freezing front (0° isotherm) descending from the ground surface mimics the morphology of the stone-soil interface because it descends faster in overlying stone regions (which are dry) than in fine-grained soils [which retain substantial water and must freeze as well as be cooled (19–21)]. Consequently, where the interface is inclined, frost heave (which acts normal to the freezing front) pushes soil down and toward soil-rich regions and pushes stones up and toward stone-rich regions, eventually giving rise to distinct stone and soil domains (11, 22).

The second feedback, between stone domain morphology and stone transport, stabilizes and promotes elongation of linear stone domains by transporting stones along their axes. Laterally directed frost heave near the stone-soil interface squeezes the stone domain (23), thereby elevating its surface by an amount proportional to lateral frost heave. Resulting along-axis gradients in uplift drive stone transport along the stone domain if stones are laterally confined within stone domains. Such confinement is promoted by low surface relief across stone domains, which results when rapid freezing of stone domains causes uniform lateral frost heave with depth in surrounding soils (24). Squeezing and confinement stabilize the vertical thickness of stone domains, because uplift increases with thickness, causing stones to avalanche from regions of high to low thickness. Similarly, squeezing and confinement stabilize the width of stone domains because wider sections, which are deeper and more easily deformed (25), experience greater uplift than do narrower sections. Squeezing and confinement also elongate stone domains because uplift promotes avalanching of stones toward and off narrow and shallow ends.

In a numerical model implementing these feedbacks (26), stones move in two dimensions representing an active layer in plan view (27). The effects of soil domains on stones are calculated from the current configuration of stones. Beginning with a random configuration, the two feedback mechanisms drive incremen-
tal stone displacements over repeated iterations, each of which represents a freeze-thaw cycle. Additionally, during each iteration, surface stones are displaced downslope a distance proportional to the hillslope gradient.

Lateral sorting is abstracted by first calculating a surface, $H_s$ that decreases with local stone concentration averaged over a radius $D_s$ (28) and weighted by inverse distance, which represents a smoothed version of the stone-soil and air-soil interfaces. Then stones are moved a distance $\delta x_s$ downslope (toward regions of high stone concentration) proportional to the local gradient of this surface $[\delta x_s = K_{s} \text{ grad } H_s]$, where $K_s$ is a diffusion constant (29) determining the rate of stone motion. Far from a stone domain, these displacements represent transport by surface creep; close to a stone domain, they represent the combined effects of surface creep and sorting caused by frost heave at freezing fronts inclined to the stone-soil interface. This abstraction simulates the positive feedback of lateral sorting because areas of high stone concentration generate dips in the surface that attract more stones.

Within stone domains, motion by lateral squeezing and confinement is abstracted as diffusion of stones biased parallel to the axis of the stone domain: $\delta x_s = K_{s} \text{ grad } U \text{at}$, where $U$ is the surface uplift owing to lateral squeezing of the stone domain (26) and $K_{s}$ is a diffusion constant (29) determining the rate of downslope stone transport. The direction of transport, $\text{at}$, is the average of a unit vector pointing along the axis of the stone domain (determined over a distance $D_s$ and weighted by a constant factor $C_s$) and a randomly oriented unit vector (weighted by the factor $1 - C_s$). The length scale $D_s$ corresponds to the distance over which the direction of lateral frost heave varies, as controlled by heat conduction and the thickness of the frozen layer (30). The nondimensional weighting $C_s$ (ranging from 0 to 1) encapsulates the degree of confinement of stones to the stone domain; increasing $C_s$ increases the along-axis component of stone diffusion and decreases the radially symmetric component. Increasing $K_{s}$ represents increasing lateral squeezing and uplift, which increases the along-axis and radially symmetric components of stone diffusion. Changing the along-axis component of stone diffusion independent of the radially symmetric component can be accomplished if $K_{s}$ and $C_s$ are varied simultaneously, keeping $K_{s} (1 - C_s)$ constant.

As the mean concentration of stones, the hillslope gradient, and the degree of lateral confinement varied in our model, sorted circles, labyrinths, islands, strips, and polygons emerged (Fig. 3). Without lateral confinement ($C_s = 0$) and as stone concentration was decreased, sorted circles transitioned to labyrinths at $\sim 1000$ stones/m$^2$ and then to stone islands at $\sim 700$ stones/m$^2$ (Fig. 3A), because isolated stone domains coalesce when separated by a distance less than the length scale associated with subsurface soil motion by frost heave, $D_s$.

With stone concentration fixed at $\sim 100$ stones/m$^2$ and increasing hillslope gradient, stone islands transitioned to downslope stripes at $\sim 10^\circ$ (Fig. 3B). This transition is determined by the magnitude of downslope transport away from a stone domain versus transport toward the stone domain by lateral sorting processes, and consequently it occurs at increasing hillslope gradient as lateral sorting into stone and soil domains progresses. Stone islands transitioned to sorted polygons with increasing lateral confinement ($C_s$, Fig. 3C) or along-axis transport ($K_{s}$). For $C_s > 0.6$, the outward transport of stones owing to lateral squeezing exceeded the inward transport of stones by lateral sorting processes, and stone islands were drawn out into the linear stone domains of sorted polygons.

Physically, the transition from islands to polygons can be attributed to decreased soil compressibility and rapid freezing in stone domains with large air-cooled pores (large stones). Less compressible soil reduces the lateral sorting mechanism that causes instabilities in the depth of stone domains. Rapid freezing increases lateral frost heave at depth, which increases squeezing and reduces the surface relief across stone domains (24), thereby increasing confinement.
Sorted polygons (Fig. 4) experience richer dynamics than other patterns because they result from an interplay between the two feedback mechanisms. Three-way intersections became equiangular in our model because frost heave-induced squeezing was reduced and less focused in the soil domain bordered by the stone domains forming the smallest intersection angle. Therefore, stones preferentially avalanching toward the smallest intersection angle, causing the intersection to migrate in this direction, thereby increasing the smallest intersection angle at the expense of the other two. This mechanism also causes four-way intersections to be unstable. At the intersection, lateral squeezing from frost heave in the soil domains encased by the two (generally opposing) larger intersection angles is greater than that in the other two soil domains; therefore, stones are squeezed between the larger soil domains and consequently avalanche toward the two soil domains enclosed by the smaller intersection angles. This divergent stone avalanching from the intersection elongates the four-way intersection into a linear stone domain with a three-way intersection at each end.

The initial spacing of sorted polygons generally was two to three times the radius over which stone concentration was calculated, \( D_{50} \). The mean polygon size increased by elimination of polygons with fewer sides than the polygons that surrounded them. The tendency of intersections to move in the direction of the smallest intersection angle causes polygons with fewer sides (and therefore smaller intersection angles) to shrink to four-way or five-way intersections that then transition to three-way intersections (Fig. 4). Similar transitions in soap bubbles and magnetic fluid froths have been reported (31, 32).

Large soil domains were dissected when random perturbations on the stone-soil interface developed into linear stone domains that extended across the soil domain, a process that was particularly active when lateral confinement was only moderate \((C_{sq} < ~0.8, \text{Fig. 3C})\) or if stone diffusion by lateral squeezing was large relative to lateral sorting \((K_{sq} \gg K_{is})\). The mean polygon size stabilized when the frequencies of dissection of a soil domain and elimination of a soil domain were similar, generally in the range \(~3D_{is} \) to \(~5D_{is} \).

Sorted polygons in the model and in nature appear similar because of the prevalence of roughly equiangular three-way intersections surrounding nearly equidimensional polygons with a fairly narrow distribution of sizes. As a quantitative test of this model, distributions of intersection angles and normalized polygon areas predicted from the model (using parameters as in Fig. 4) were compared with the corresponding distributions measured from sorted polygon networks within two desiccated pond basins in Alaska (Fig. 5 and Table 1) (33). Within the variability between the measured networks, modeled and measured polygons are consistent.

In our model, all forms of sorted patterned ground form via self-organization from just two straightforward feedback mechanisms: lateral sorting and stone domain squeezing. Sharp transitions between patterns occur as three parameters are varied: stone concentration, hillslope gradient, and the relative strength of lateral sorting and squeezing. Squeezing and confinement—which are critical for the development of the most common pattern, sorted polygons—are enhanced by rapid freezing in stone domains with large, air-cooled pores and by low soil compressibility. This model suggests that the presence and type of pattern can be diagnostic of active layer properties and the relative magnitudes of the primary transport processes.
Fig. 5. (A) Intersection angle and (B) polygon area (normalized by the mean polygon area) probability distributions from sorted polygons in Alaska and predictions from the model using parameters as in Fig. 4, except with simulation size of 30 × 30 m. Dashed line, west pond; dashdot line, east pond; solid line, model. Error bars represent SD of 10 independent model runs. Model is consistent with measurements within their level of variability.

References and Notes
4. S. Taber, *J. Geol.* 37, 428 (1929).
22. The lateral sorting feedback is consistent with the limited range of relevant field observations. Frost-susceptible soils overlain by surface stone layers often exhibit lateral sorting associated with soil plugs rising to the surface (34). A model for sorted circles based on this lateral sorting feedback is quantitatively consistent with field measurements (11).
23. Observations within sorted polygons of upended stones, stones aligned parallel to the stone domain axis, and mud folds parallel to the stone-soil contact are consistent with lateral squeezing of stone domains (35, 36).
24. Surface relief across stone domains reveals removal of soil from beneath frost heave at the stone-soil interface. Because stone domains narrow with depth, frost heave near the ground surface removes soil beneath the edge of the stone domain, whereas frost heave at depth removes soil beneath the center. In steady state, surface gradients are balanced by stone avalanching and removal of soil is balanced by soil addition processes whose impact on elevation is distributed across the stone domain surface.
25. Lateral squeezing is dependent on stone domain width (relative to stone diameter) owing to increased resistance to deformation by narrower stone domains.
26. See supporting data on Science Online.
27. Unlike previous simulation models (10, 11, 37), stone position is not discretized, allowing congestion effects that can cause trapping and jamming of stones on lattices. However, here a lattice with grid size corresponding to stone diameter is used to calculate the concentration of stones and to determine the depth to which they are stacked. This semicontinuous algorithm allows for greater sensitivity to local biases because stones can move in any direction and over distances less than a cell width. Stones can also overlap, representing stacking of stones, up to a limit corresponding to the maximum depth of freeze-thaw processes.
28. The length scale D_s corresponds to the maximum lateral distance over which soil is displaced by frost heave (11).
29. K_0 and K_w are likened to diffusion constants because they quantify the ratio of stone flux to gradient.
30. The magnitude of squeezing depends on the thickness of the ice-rich layer along the near-vertical boundaries between stone and soil domains. Squeezing can be influenced by soil domain size for small soil domains, but well-formed soil domains generally are much larger than the thickness of the ice-rich layer. Therefore, this dependence is not included in the model (i.e., D_s does not depend on soil domain size).
33. Measured sorted polygons were located in the basins of two desiccated ponds 100 m north of Denali Highway, ~115 km east of Cantwell, Alaska. Both ponds exhibit a gradation from high to low stone concentration moving outward from their centers. Well-formed polygons with narrow stone domains located midway in this gradation were digitized from low-elevation aerial photographs that had been orthorectified using ground control points (38). East pond: 68 polygons, 436 angles. West pond: 155 polygons, 705 angles.
39. We thank A. B. Murray and B. Hallet for many helpful discussions, and L. Clarke for assistance in image processing. Supported by NSF Arctic Natural Sciences Program grant OPP-9530860, the Andrew W. Mellon Foundation, a National Defense Science and Engineering Graduate Fellowship, and a student research grant from the Whole Earth Society at Scripps Institution of Oceanography.

Supporting Online Material
www.sciencemag.org/cgi/content/full/299/5605/380/DC1
SOM Text
Fig. 51

13 August 2002; accepted 19 November 2002

The Global Morphology of Wave Poynting Flux: Powering the Aurora

Large-scale, electric currents flowing along magnetic field lines into the polar regions of Earth are thought to be the main contributors of the energy that powers the ionospheric aurora. However, we have found evidence for global contributions from electromagnetic waves (Alfvén waves). Data that were collected from the Polar spacecraft over the course of 1 year show that the flow of wave electromagnetic energy at altitudes of 25,000 to 38,000 kilometers delineates the statistical auroral oval. The Poynting flux of individual events distributed along the auroral oval was larger than 5 ergs per square centimeter per second, which is sufficient to power auroral acceleration processes. This evidence suggests that in addition to magnetic field-aligned currents, the dayside and nightside aurora is globally powered by the energy flow of these high-altitude Alfvén waves.

Earth’s aurora occurs statistically and often simultaneously in an oval-shaped belt (Fig. 1A) around the magnetic poles (1). Magnetic field lines connect this auroral oval to the magnetosphere, the region above the atmosphere that is dominated by Earth’s magnetic field and filled
as many as one-third of exotic strains may also show efficient oral transmission.

The authors argue that the fateful cross producing the three predominant clonal lineages may have brought together genes promoting efficient transmission by the oral route and perhaps other necessary adaptations as well. In almost all genetically diverse organisms, recombination can produce progeny with traits differing significantly from those of either parent. For example, a laboratory cross between type II and type III strains of T. gondii produced some progeny that were 1000 times as virulent as either parent (2). (A few percent of natural isolates of T. gondii are in fact recombinants between the clonal lineages.)

When did the fateful cross take place? To address this issue, Su et al. studied polymorphisms in noncoding DNA sequences (primarily introns) in 10 isolates of the predominant strains from diverse geographical locations. On the assumption that nucleotide mutations in noncoding DNA are selectively neutral (or nearly so), the number of unique polymorphisms that have arisen in each lineage since the time of the original cross is expected to increase in proportion to the mutation rate. Across 4067 base pairs in each of the 10 strains, the authors found only two new mutations. If the mutation rate in T. gondii is similar to that in the apicomplexan malaria parasite Plasmodium falciparum, then this result implies that the predominant clonal lineages diverged about 10,000 years ago. This estimate contrasts sharply with the time of divergence for exotic lineages (about 1 million years ago).

About 10,000 years ago, human society was undergoing one of its epochal transitions, from hunting and gathering to slash-and-burn agriculture. The population density was increasing, creating new opportunities for endemic or epidemic transmission of many kinds of parasites, and the cat was adopted as a companion animal. These conditions could well have favored strains of T. gondii that traded frequent sex in favor of rapid and efficient oral dissemination.

The finding of little genetic variation in each of the three predominant clonal lineages has important implications for public health. It means that drug resistance or immune evasion in these lineages must be acquired through the occurrence of new mutations, rather than through the selection of rare mutations that already exist. The waiting time for such mutations to occur may be substantial. On the other hand, the large amount of genetic variation present in the exotic strains, as well as in the recombinants between the predominant strains, implies that any sudden increase in the frequency of these strains should be viewed with alarm.

References

GEOMORPHOLOGY

On Patterned Ground

Daniel Mann

 Intricate patterns of stones and mud (see the figure) decorate the ground in many cold landscapes. Part of the beauty of this patterned ground comes from the contrast between its stony geometry and the disorder of its bleak surroundings. On page 380 of this issue, Kessler and Werner (1) venture an explanation for how the patterns form and maintain themselves. Building on their earlier research on landforms as diverse as sand dunes and beach cusps (2–5), they use computer simulations to model patterned ground as a self-organizing system. The approach may help to understand the origins of the bewildering variety of patterned ground in alpine and polar regions (6, 7).

Geomorphologists have long been preoccupied with how patterned ground develops in cold regions (8). Nordenskjöld (9) suggested that the patterns resulted from aqueous convection currents in the ground caused by temperature differences, whereas Nansen (10) thought that they arose from the ejection of stones by freezing. Explorers, geologists, botanists, and native peoples all had their favored explanations. Laboratory and field studies have validated some mechanisms (11–13), but no general explanation has emerged.

Patterns on the ground. In polar and alpine environments, stones and mud often form patterns such as stripes and polygons, depending on the ground slope.

Washburn (6) predicted almost 50 years ago that a general explanation of patterned ground would involve two mechanisms: the compression of the stone domains by expansion of the adjacent soil during freezing, and the upfreezing of stones. But it has remained unclear how these mechanisms interact to create the often highly ordered, three-dimensional (3D) phenomena of patterned ground. Kessler and Werner now add a third mechanism, the gravitational redistribution of heaved stones. They show that acting over centuries, these three processes can together cause the development of self-maintaining landforms wherever the ground repeatedly freezes and thaws.

Although the physics of these mechanisms are simple, two obstacles have prevented us from understanding how they create patterned ground. The first is a problem of approach: At what spatial scale should we look for an explanation? Most researchers have looked for explanations in the physics affecting the individual grains of silt, sand, and pebbles that form the patterns. Unfortunately, this
familiar Reductionist approach has never worked very well for patterned ground. The second obstacle is related to the first one but is purely technical: How can the 3D effects of repeated freeze-thaw cycles on a given volume of rocky soil be simulated over hundreds of years? Kessler and Werner solve the conceptual problem by applying a self-organization approach (14) that takes into account spatial and temporal scales greater than those of the individual stones and sand grains. They couple this approach with computer simulations that track the movements of thousands of individual stones within a field of cyberspace tundra (7).

Kessler and Werner's simple model answers a question as basic as "Why is the sky blue?" and makes sense with what is seen on the ground in polar or alpine environments. The work is also exciting because it embodies a new point of view that is affecting the entire field of geomorphology (the subdiscipline of geology that others have enviously described as the "science of scenery"). The field is experiencing a paradigm shift from a reductionist approach toward concepts such as universality and self-organization (15).

Reductionism assumes that all characteristics of geomorphic features, from ripples to sand seas, can ultimately be predicted from first principles applied to fundamental particles. The geomorphic phenomena on Earth's surface are then the mere by-products of much smaller scale processes. Self-organization offers a different viewpoint (16). Landforms are products of self-assembling hierarchies of processes. Interacting groups of mechanisms, like the three discussed above for patterned ground, are linked by feedbacks that span a range of spatial and temporal scales. In Kessler and Werner's model, the positions and contours of the separate domains of fine and coarse particles—which have developed over decades to centuries and have length scales on the order of meters—fluence the freezing and thawing of ice lenses located within each square centimeter of the soil domain. These lenses freeze and thaw over hourly to monthly time scales, and it is their orientation and amount of heave that maintain the overall landform.

Hence, smaller, faster processes are enslaved by larger, slower ones within the patterned ground system. Such interactions can have unexpected results, and self-organizing systems typically have emergent properties that are not predictable from the physics of their fundamental particles. There is nothing in the physics of a shovelful of stony mud that can predict the emergence of an intricate pattern of interlaced, stone-bordered polygons covering many square meters. According to the self-organization paradigm, many geomorphic phenomena on Earth's surface are responsible for their own development and maintenance. A landform is not just a by-product of processes operating at the scale of its fundamental particles; it can only be understood at its own greater-than-sand scales.

There are many more types of patterned ground than those investigated by Kessler and Werner, and no single model can explain them all. But their approach gives us a new investigative tool to try out on other patterned features. Of course, models must be treated with caution, because they can mimic the work of nature but use the wrong mechanisms. Also, similar types of patterned ground can arise in different settings as a result of quite different mechanisms. Some types of patterned ground have very simple forms, and the simpler the form, the easier it is for different processes to create them.

The self-organization perspective of Kessler and Werner (1) paper brings up some interesting questions. If self-organized entities are widespread in Earth's most desolate environments, are the milder climes teeming with them unnoticed? Is self-organization as inevitable as gravity? Self-organization entails self-making and self-maintaining, and these are characteristics of living things. So where is the division? And do self-organized entities compete with each other for growing space and for the energy flows that sustain them? For instance, do sorted circles and polygons somehow fight it out for possession?

References
11. S. Taber, J. Geol. 37, 428 (1929).